Preserving Long-term Variability in a Daily Rainfall Model using Stochastic Parameters of Markov Chain and Gamma Distribution

AFM Kamal Chowdhury, Natalie Lockart, Garry Willgoose, George Kuczera, and Anthony S. Kiem
The University of Newcastle, Australia

SMART FUTURE CITIES 2015 — Energy, Transport & The Built Environment Innovations for Future Cities — NEWCASTLE, 1–3 OCTOBER

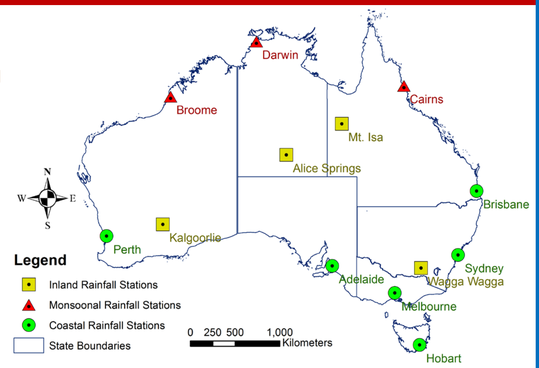
THE UNIVERSITY OF NEWCASTLE AUSTRALIA

Introduction

The daily rainfall models often underestimate the long-term rainfall variability in monthly to multi-year timescales. To resolve this issue, we have developed a new model with improved parameterization techniques of traditional 'Markov Chain (MC) – Gamma Distribution' approach. We used the transition probabilities of dry-to-dry and wet-to-wet days (MC parameters) for rainfall occurrence simulation, and mean and standard deviation of wet-day rainfall (Gamma parameters) for rainfall amount simulation. But rather using a single set averaged parameters, we calculated the parameters for each month of each year. Then, we fitted a truncated normal distribution to the observed MC parameters and stochastically sampled them in the model. If there were statistically significant monthly autocorrelation in the observed MC parameters, the stochastically sampled MC parameters were modified using a lag-one autocorrelation equation and then used to simulate the rainfall occurrence. We also fitted a bivariate-lognormal distribution to the gamma parameters and stochastically sampled them in the model to simulate rainfall amount in wet days. While other existing models are either very complex or do not preserve the long-term variability, our model uses the simple technique of stochastic parameterization to preserve the long-term rainfall variability.

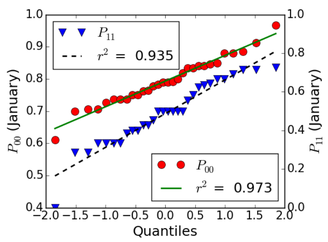
Rainfall Stations and Data

We used ground-based daily rainfall data for 1979-2008 (30 years) period from 13 Bureau of Meteorology raingauge stations in three climate zones of Australia.

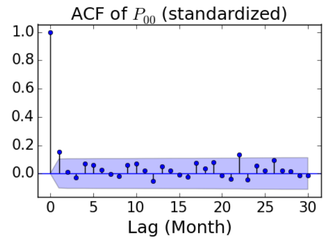


Markov Chain Parameters

Normal distribution (truncated between 0 and 1) of dry-to-dry (P_{00}) and wet-to-wet (P_{11}) probabilities for January in Brisbane



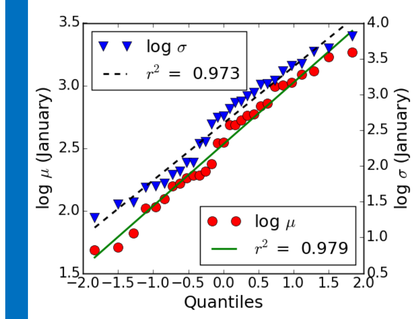
Autocorrelation Function (ACF) of month-to-month P_{00} values in Brisbane



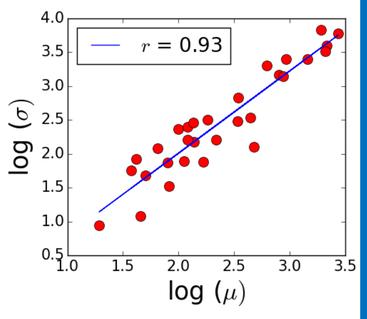
- Sample MC parameters from their truncated normal distributions
- If lag-1 ACF is significant, modify sampled parameters using a lag-1 autocorrelation equation

Gamma Parameters

Lognormal distribution of mean (μ) and standard deviation (σ) of wet day rainfall for January in Brisbane

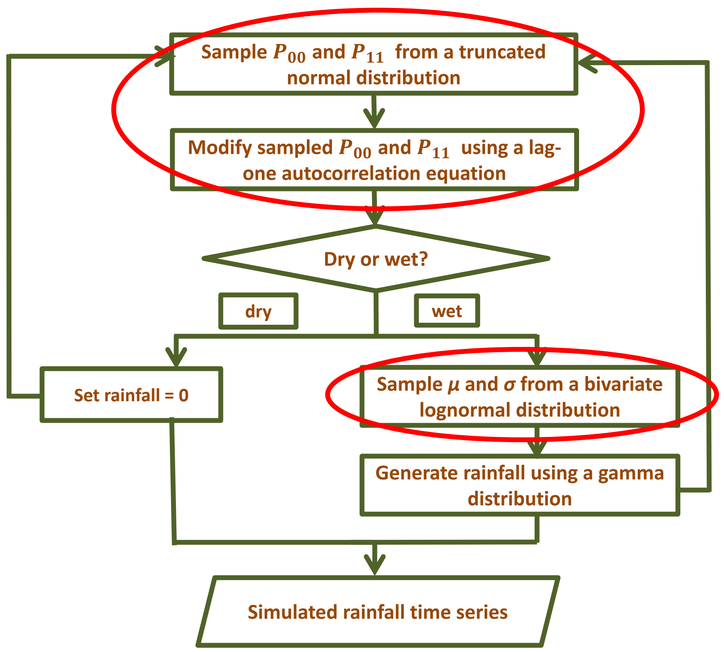


Strong correlation between log μ and log σ values



- Sample μ from lognormal distribution and σ from bivariate-lognormal distribution

Model Schematic



- The model operates at daily time-step with varying parameters in each month
- = Major Contribution of this Study

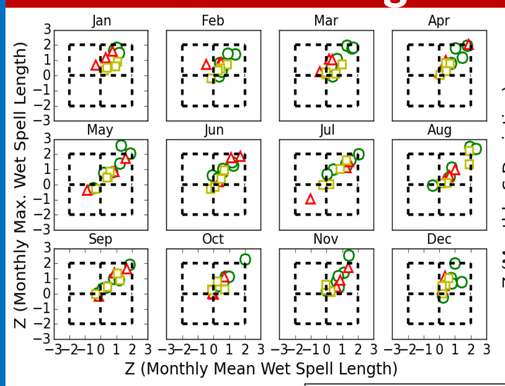
Model Assessment

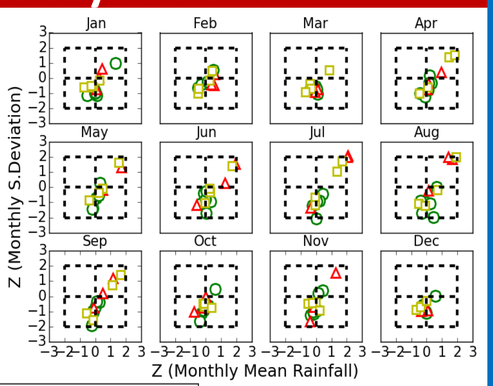
For each station, the model was run 100 times with each run being 30 years long. In each run, we calculated some desired statistics in different timescales. Then, for each statistic, we calculated the mean (expected value) and standard deviation of the 100 realizations, and calculated the Z score of the statistic as follows:

$$Z = \frac{\text{Observed value} - \text{Expected value}}{\text{Standard Deviation}}$$

Z between -2 and +2 ➡ observed statistic is within 95% confidence limit of the model simulations

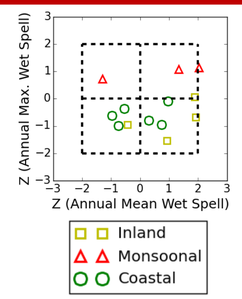
Preserving Monthly Statistics

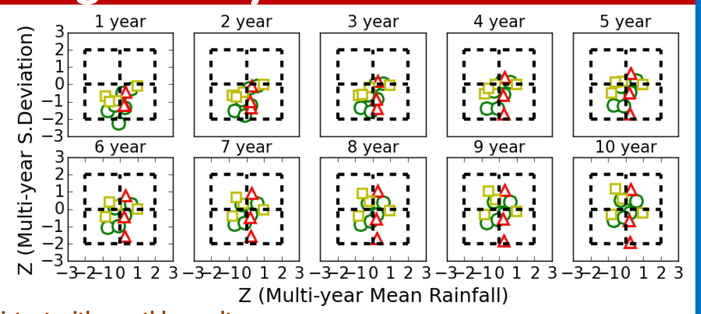




- Monthly rainfall amount and wet spell statistics are preserved well in the model with most of the observed statistics fall within the 95% confidence limit of the simulations (-2 $\leq$ Z $\leq$ +2)

Preserving Multi-year Statistics





- Multi-year results are consistent with monthly results
- The model has performed satisfactorily in all three climate zones

Conclusion

This study developed a daily rainfall model using stochastic parameters of Markov Chain and Gamma distribution. The model was calibrated and assessed using raingauge stations data from three climate zones (Inland, Monsoonal and Coastal) of Australia, which are representative of most of the world climate zones. We found that the key rainfall occurrence and amount statistics in monthly and multi-year scales were well-preserved (mostly within 95% confidence limit) in the model. Therefore, this model is potential to be a simple alternative to other complex models to be used in agricultural and hydrological planning and design, where preserving long-term variability is a concern.